Canonical coin systems

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April 14, 2016

Overview

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   - Properties of subvectors
   - $M(w)$ vs $G(c_{i-1} - 1)$
3. Polynomial algorithm for checking canonicity

Brief description of the problem

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Denominations

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14

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14

Greedy

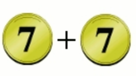
Brief description of the problem

Denominations

14

Greedy

Optimal



Definitions

$C = (c_1, c_2, ..., c_n)$

where $c_1 > c_2 > ... > c_n$ are denominations

Decomposition of w

vector V satisfying $V \cdot C = w$ (where $\cdot$ is scalar product)

$G(w)$

greedy decomposition of w

$M(w)$

optimal decomposition of w (minimalize $|M(w)| = M(w) \cdot (1, ..., 1)$)

Properties of greedy decomposition

$G(\cdot)$ is monotonic

$x \le y \implies G(x) \le G(y)$

$G(w)$ is the greatest lexicographical representation of w

$\forall_{U : U \cdot C = w} U \le G(w)$

Properties of subvectors

Definition

V *is greedy* if and only if $V = G(V \cdot C)$

Definition

V *is minimal* if and only if $V = M(V \cdot C)$

Lemma

a) If $U \subseteq V$ and V is greedy, then U is greedy.
b) If $U \subseteq V$ and V is minimal, then U is minimal.

$M(w)$ vs $G(c_{i-1} - 1)$

Theorem

Let w be the smallest counter example, i - the first index of nonzero value in $M(w)$, j - the last one. Then $M(w)$ is identical with $G(c_{i-1} - 1)$ on positions from 1 to $j - 1$-th. It's one greater on j-th position. The remaining entries are all zeros.

$O(n^2)$ candidates

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every checked in $O(n)$

$O(n^2)$ candidates

every checked in $O(n)$

$$=$$

$$O(n^3)$$

Thank you