

## 1 $M(w)$ and $G(w)$ don't have common nonzero entries

Subtracting one on that position produces smaller counterexample.

## 2 $c_{i-1} \leq w$

$G(w)$  has to have nonzero entry before  $i$ -th position because  $M(w) < G(w)$  and  $M(w)_k = 0 \forall k < i$  and  $M(w)_i \neq 0$  and  $G(w)_i = 0$ .

## 3 $w - c_j < c_{i-1}$

Subtracting one from minimal representation  $M(w)$  on  $j$ -th position gives minimal representation  $M(w - c_j) = G(w - c_j)$ . It doesn't have nonzero entries before  $i$  so  $w - c_j < c_{i-1}$  (greedy algorithm).

## 4 $G(c_{i-1} - 1) < M(w)$

Subtracting one from both vectors on  $i$ -th position gives still greedy and minimal representations respectively of  $c_{i-1} - 1 - c_i$  and  $w - c_i$ . The second one is also greedy (smaller than smallest counterexample). But we know that  $c_{i-1} - 1 - c_i < w - c_i$  so  $G(c_{i-1} - 1 - c_i) < G(w - c_i)$ . Now we can come back, adding one on  $i$ -th position.

## 5 $M(w - c_j) \leq G(c_{i-1} - 1)$

$$M(w - c_j) = G(w - c_j) \leq G(c_{i-1} - 1)$$